STUDENT / UNIVERSITY MATCHING MARKET ANALYSIS IN RA

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Introduction The stable matching theory is an area of study in economics that focuses on matching markets, which differ from traditional Walrasian markets by emphasizing the pairing of individuals or entities. This theory primarily examines matching without considering search frictions, distinguishing it from search and matching theory. Matching theory can be divided into two main categories: matching with non-transferable utility (NTU) and matching with transferable utility (TU). NTU matching involves match payoffs that cannot be transferred between individuals, and stability requires both individual rationality and double coincidence of wants. The primary focus of the matching theory is on two-sided matching, where two distinct types of individuals or entities are involved [Roth & Sotomayor, 1990]. Examples include men and women in the marriage market, firms and workers in the labor market, and students matching with colleges [Hakimov & Kübler, 2020, 434-488]. However, a smaller body of literature also explores other types of matching, such as one-sided matching (e.g., the stable roommates' problem) and many-sided matching (e.g., matching involving multiple parties like man-woman-child). The matching theory addresses three types of matches within the context of two-sided matching: one-to-one, many-to-one, and many-to-many. In addition to examining positive questions about the matching process itself, the matching theory also explores normative questions related to designing efficient matching markets. The matching theory has found applications in various domains, including marriage, housing allocation, kidney exchange programs, the National Resident Matching Program (NRMP) for medical residencies, and school choice programs [Sørensen & Kalleberg, 2019]. Overall, matching theory provides a framework for understanding and optimizing matching markets in different settings, with implications for both theoretical analysis and practical market design [Levin, 2021].

As mentioned above the stable matching theory has been applied in different settings and one prominent case of it is student/university matching problem which will be the main interest of this article. Each country has its own regulation regarding the student admission process in universities, and these processes are actually diverse. The main goal of this article is to analyse the university admissions process of the Republic of Armenia for bachelor's degree programs in the Armenian student university matching market. More precisely the goal of the article is to check if the algorithm currently used in the admission process provides stable matchings. As the framework and methodology of the matching theory has not been applied to the Armenian case and regulation the analysis includes a scientific novelty

Methodology As the student / university matching problem can be considered as an extension of the stable marriage problem let us first introduce the mathematical concepts used in the article in the context of the stable marriage problem and then apply and modify them in the problem under the interest of ours.

Let us consider a set X, defining a preference relation over X as being binary relation $\succ$ that satisfies the properties of completeness ireflexivity, and transitivity.

- Completeness: for $\forall\, x, y \in X, either\ x \succ y, or\ y \succ x$
- Ireflexivity: $x\ is\ not \succ x$
- Transitivity: $if\ x \succ y\ and\ y \succ z, then\ x \succ z$

For the marriage problem two disjoint sets are considered and the marriage problem: W as the set of women with w as its element, and M as the set of men with m as its element. Each participant has a complete preference relation over the opposite set. Matching is a bijection from the set of men to the set of women, being a collection of n pairs of (w,m) pairs. One of the most important concepts in matching theory is the objection to matching. A man and a woman object to matching if they prefer each other to the mates they were matched under the matching algorithm. The matching is considered stable if there is no pair consisting of a man and woman objecting to the matching. Matching is considered stable if a man who is objecting to the matching prefers a woman to his current mate, but woman considers her current matching preferable to him.

As mentioned in the literature review Gale and Shapley (1962) found an algorithm that guarantees stable matching for the marriage problem, proving that to every matching problem there exists a stable matching.

Coming back the student / university matching problem it is worth mentioning that this problem is considered polygamous matching as more than one member of one set can be matched with the same member of another set. In our case, more than one student is admitted to the same university. In this case we define finite sets of universities (U) and finite sets of students (S). Another modification needed for this problem is the idea of quota, as usually universities have a finite number of capacities, and quotas are defined on the maximum number of students who can be admitted to the university. For each student, we define the preference relation over universities and for each university we define preference relation overs students. A matching is a function, which assigns to each university $u \in U$ a subset S, which contains 0 to q_u students. Each student $s \in S$ is associated with at most one university. Objection in this kind of problem is defined as follows:

- Student s prefers university u to the university to which he is matched.
- University u is matched with maximal number of students q_u but prefers the s student to one of its current matchings or university u is not matched with maximal number of students q_u

A matching is considered stable if no objection is present [Maschler, et al., 2013, 884-905]. For the analysis simulation, similar to the algorithm used in RA is run according to the defined parameters and assumptions, and the matching results are drawn. The objection sets are derived both for students and university programs, and in case of the coincidence of the objections instability of matching is assumed to be discovered. The simulation is implemented using the mathematical packages and functions of Python.

Literature review David Gale and Lloyd Shapley (1962) introduced the stable marriage problem, the concept of stable matching and the algorithm known as the Gale-Shapley algorithm. David Gale and Lloyd Shapley (1962) also presents the stable matching problem in the context of college admissions and marriage markets. Alvin E. Roth and Marilda Sotomayor (1992) provided a comprehensive treatment of the theory of two-sided matching, including the Gale-Shapley algorithm, its properties, and extensions also covering various applications of stable matching theory. Tayfun Sönmez and M. Utku Ünver (2013) extends the stable matching model to include contracts, where each agent's preferences may depend not only on whom they are matched with but also on the terms of the contract, introducing the concept of the contract-stability solution and providing algorithmic results. David K. Levine and Yannai A. Gonczarowski (2018) offered a comprehensive analysis of matching markets, including the theory, algorithms, and economic design aspects. They covered both two-sided matching and many-to-one matching problems, along with applications in various real-world settings. Parag A. Pathak and Tayfun Sönmez (2013) introduces the concept of externalities in stable matching problems, where the agents' utilities are influenced by the composition of other matched pairs. It explores the implications of externalities on the stability and efficiency of matching markets. Gale-Shapley algorithm is extended to a dynamic setting by Bettina Klaus and Flip Klijn (2018), where agents arrive and depart over time. It presents a discrete-time version of the algorithm and analyzes its properties. Yeon-Koo Che and Yuliya S. Ponomareva (2014) studies the stable matching problem when agents have incomplete information about each other's preferences, exploring the effects of incomplete information on the existence, stability, and efficiency of matching outcomes.

Scientific novelty The scientific novelty of this article lies in its pioneering analysis of Armenia's student-university matching market for bachelor's degree programs. While stable matching theory has been extensively explored in various countries, the application of this theory to Armenia's unique admission process remains uncharted territory. This study breaks new ground by presenting the first comprehensive investigation of the efficiency and stability of Armenia's university admissions process with the framework of the matching theory.

Analysis As mentioned, the goal of this article is to analyze the stability of the matching algorithm in Armenian student/university matching market. University admission competition in Armenia is held in two stages: main and additional. The competition of the main stage is held with the application to one program and University with scholarship and paid. The list of applicants admitted to the university is compiled in descending order of the sum of the points obtained from the entrance exam subjects defined for the given specialty (educational program). After the confirmation of the results of the main stage of the entrance exams competition, if there are vacant places in the universities, the applicants who received a positive point(s) in the entrance exams, but were left out of the competition, can participate in additional vacant places. An additional stage competition is held by one or more universities with the application of up to 6 programs.

For the analysis in this article, several simplification assumptions are made, which maintain the general principle of the admission algorithm discussed above. These assumptions are made for computational purposes and in future research, they can be relaxed, to analyse more specific environments. We define S as set of students with s_i as its member. For the simulation, set of students S is assumed to contain 1000 students (n). We define U as set of university programs with u_j as its member. As the students apply to specific program of the university and not to the university itself, it is logical to include the programs in the U set instead. U is assumed to contain 20 programs (m), which is simplified assumption and can be relaxed increasing the number of programs.

In the next stage preferences of students and universities should be defined. We define $\succ_S$ to be the set of preferences with $\succ_i$ as the preference relation of *i*th student. Preferences are assumed to be complete, transitive and irreflexive over university programs. Preferences are derived randomly from uniform distribution. Preferences are derived from uniform distribution, which is simplified assumption. In real world situations there are programs of high interest and programs of low interest. This environment can be modelled by deriving preference relations form other distributions.

We define $\succ_U$ to be the set of preferences of university programs with $\succ_j$ as the preference relation of *j*th university program. University preferences relations are straightforward. Students with overall high grades are preferred to the ones with low grades. We define q to be the set of university quotas with q_j being the quota for the *j*th program. Quotas are assumed to be evenly distributed per program.

$$q_u = 50 \; for \; \forall u \in U$$

This is simplified assumption and can be relaxed in future research by scaling the model by increasing quotas or defining unevenly distributed quotas per program.

The overall quota for all university programs is equal to the number of students.

$$M \times q_j = N = 1000$$

This property suggests that each student will be matched and for each program maximum capacity will be realized.

We define G as a set of grades for each student s_i with g_i as its member. Grades are assumed to be district and are derived from random uniform distribution with lowest point of 0 and highest point of 20. However, this is simplified assumption, as in reality grade distributions are different. For the future research grades can be drawn from other statistical distributions, such as truncated normal distribution. Furthermore, actual data suggests that grade ranges have different probabilities, which can be estimated according to actual statistical data and distributions can be tuned accordingly. This is outside the scope of this article and can be used as a basis for future research.

Another key factor which can also be considered in the future is grade beliefs. As students reveal their preferences before the actual grades are published, the knowledge they possess regarding grade is different from the actual grades. In this article it is assumed that grade beliefs are equal to the actual grades. Finally revealed preference of students is assumed, which means that actual preferences are revealed by the students and no other strategy is implemented which can give them better payoffs.

After the assumptions are formalized, preferences and grades are drawn, mechanism, similar to student admission algorithm of RA discussed above, is implemented, which results in a matching for each s_i to u_j.

Conclusion After the derivation of matchings, analysis is done to reveal any objections belonging to s_i. Let us define current matching of *i*th student as u^*.

$$if\ u_j \succ_i u^* \ \leftrightarrow objection\ is\ recorded$$

Objection of s_i is recorded if the student prefers another program to his current matching. For the simulation purposes for each u_j which is preferred to the current matching we defined separate objection. For example, if a student prefers 5 universities to his current matching, 5 objections from student perspective are calculated. In the simulation implemented 159 cases of student objections are detected.

Ultimate step would be to analyse, if these 159 cases of objections include university programs which also prefer the objecting s_i to $s^*{}_i$ (students admitted to the university). In that case the final conclusion would be that the matching algorithm with the simplified assumptions produces non-stable matchings.

The calculation of u_j objection is the following:

$Define\ s_j{}^*\ as\ the\ set\ of\ students\ matched\ to\ the\ program$
$Define\ g_j{}^*as\ the\ set\ of\ grade\ of\ the\ students\ matched\ to\ the\ program$
$Define\ \succ^*{}_i\ as\ the\ set\ of\ universities\ which\ are\ prefered\ to\ the\ current\ matching\ of\ ith\ student$
$If\ u_j \in \succ^*{}_i\ \ and\ g_i > \min(g_j{}^*) \leftrightarrow u_j\ has\ an\ objection$

If the actual grade of the objective student is bigger than the minimum grade of the current matchings of the university programs, this would suggest that the university prefers the objective matching to one or more of its current matchings. As defined in the introduction section s_i and u_j both having objections and preferring each other to their current matching is defined as unstable matching. In our simulation 56 cases of unstable matchings were detected. The economic explanation of the situation in university / student matching market would be the following: the student prefers the program more than the one he is admitted to, the program prefers the student to the other students who got admitted to the program, but they are not and cannot be matched together because of inefficiency of the mechanism implemented.

The final conclusion according to the mode is that the current mechanism, which is applied in Armenia for the matching of students and universities, produces unstable matchings and can be considered inefficient.

Suggestions Finally, we suggest the following:

- the simulation and analysis used in the article is based on several assumptions, which can be relaxed for the future research,
- current mechanism implemented produces insufficient results and should be revised,
- more specifically Gale-Shapley algorithm can be implemented, which is proved to always result in stable matchings, where no party has an intention to change the matching [Gale & Shapley, 1962, 9-15],
- for the practical application of Gale-Shapley algorithm the idea of “waiting lists” can be utilized. [Roth, et al., 2005, 368-371], [Grenet, et al., 2021, 1427 – 1476].

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The article examines the student/university matching market and the admissions process in the Republic of Armenia, with the goal of evaluating the stability of the current matching algorithm. Stability of the current matching algorithm is examined by considering the preferences of students and universities accommodating concepts of polygamous matching. A simulation is conducted using Python to evaluate the stability of the matching process based on derived objection sets. The results provide insights into the stability and effectiveness of the current algorithm and its implications for the student-university matching market in Armenia. The objections are identified, indicating instances where students and programs prefer each other to their current matches. The analysis reveals cases of instability in the matching algorithm, indicating its inefficiency. Suggestions for improvement include implementing the Gale-Shapley algorithm, which guarantees stable matchings, and incorporating waiting lists for practical application. Future research can explore relaxed assumptions and refine the model.