QUANTITATIVE ASSESSMENT OF CREDIT RISK

Narek KESOYAN
PhD in Economics, Assoc. Prof., Chair of MME, ASUE
Mesrop MESROPYAN
BA at YSU Actuarial and Financial Mathematics

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Introduction. Risk classification is one of the main functions of banking activities, as their precise structure and grouping make the planning and forecasting process more efficient. Comprehensive classification of bank risks is important on the assumption that it provides an opportunity to model and regulate banking activities, conduct a comprehensive research of internal reserves, as well as increase the efficiency of operations. It also allows to free the key performance indicators of the bank from the impact of externalities, increasing the accuracy of managerial decisions.

Currently, the hierarchical system of banking risks has a fragmented and improper structure, which is primarily conditioned by the diversity of approaches to measuring banking risks and the availability of different approaches to coordination.

Scientific novelty. The scientific novelty of the work the proposed model, which allows to assess the credit risk based on the probability distribution density function of the provided loans, according to the different scenarios of the strategies chosen by the bank. Taking into account the issue of the unified approach to the classification of the nature of the risks, the factors of origin and the source of origin, it becomes vital to take steps to create a comprehensive methodology.

The model presented in the work makes it possible to assess credit risk in case of different distribution functions as well as its sensitivity to changes in various parameters.

Literature review. Currently there are about 40 risk classification indicators, and still more than 220 types of risks typical of banking activities are being observed [Kad-zho, 2015, 51]. One of the outstanding researches on the classification of banking risks is N. Sokolinskaya's work titled as "Economic Risks in Commercial Bank Activities" [Sokolinskaya, 2000]. It highlights criteria that have not undergone significant changes over time as a result of the development of banking risk theory. One of the disadvantages of this approach is the lack of risk sharing in accordance with their importance, also the relationship of risk to different groups and the impact on each other are not shown and the elements of the classification are eclectic. The strengths of the author's classification are the systematic approach to risk analysis, the need to manage banking risks through a combination of known tools over time and space, the gross banking risk calculation algorithm and the urgency to develop banking risk theory [Sokolinskaya. 2000]. V. Sevruk classifies risks, including banking risks, in the financial sector, since "the effi-ciency of the financial sector of the country's economy has a decisive impact on the acti-vity of the economy as a whole." According to V. Sevruk, the complexity of risk classi-fication is explained by their diversity, as well as by the emergence of new types of risks as a result of economic, social and technological development of the modern world [Sev-ruk, 2001]. The classification of external risks proposed by V. Sevruk is more compre-hensive [Sevruk, 2001, 9-33], as it emphasizes the risk of the country, the risk of foreign exchange, the risk of natural disasters, etc.

M.I. Bakanova, I.O. Spitsina, O. Ya. Spitsina, IT Balabanova, N.E. Sokolinskaya and other scholars offer their own versions of risk classification. Here the main risks in-clude credit risk, market risk, interest rate risk, liquidity and imbalance risks, profit loss and insolvency risks. Additional risks are inflation, currency, political and abuse risks.

J. F. Sinki [Sinki, 1994, 937], considering the analysis of the balance sheet of the credit institution, distinguishes three main groups of banking risks: *portfolio* (credit, in-terest, foreign exchange, liquidity and capital management) and *off-balance sheet risks, regulatory risk*, delivery risk (technological, affiliation, current efficiency, strategic risk). German Economists P. Welker and B.Oldesloe distinguishe two groups of banking risks: non-payment risks and price risks. [Alisheykhova, 2006, p.15-17] distinguishes five types of banking risks: credit, liquidity, interest rate risk, operational risk, capital risk (probability of bank insolvency).

The probability of not achieving the goal can probably be considered the most suc-cessful definitions of risk amongst many existing ones, but mathematical expectation of the expected result and dispersion are often used in practice. Moreover, the last two methods of risk assessment cannot be considered sufficiently justified. Of course, it is not perfect to measure risk with the probability of not achieving the goal, especially if it is formally perceived, then, for example, achieving the goal by 99% will formally mean that we have not achieved the goal. In many cases, being close enough to the goal can also be considered achieving the goal, for example, if that goal was to achieve a certain standard of living, then achieving it by 99% can also be considered a complete success, although we did not achieve the goal from a formal point of view.

Research methodology. Toolkit of probability theory, modern systems of risk classification and management strategies are implemented in this work. Particularly, is-sues such as distribution density function of random variables, their numeric character and distribution function, the concept of function of random variables, mathematical me-thods and models of quantitative risk measurement and assessment, as well as modern risk classification systems used to conduct research in the field of risk theory, several possible types of credit policies needed to manage them in conditions of uncertainty are touched upon in the current research.

Analysis. The use of linear models, as well as models based only on the assump-tion of normal distribution, are usually applicable in cases where the system is operating in constant conditions. The interdependencies between the elements of the model object are constant over time and are not subject to the impact of various factors. However, most of the variables included in the mathematical model are random variables or functions that may change over time. When evaluating the impact of external parameters or factors on a model object, only their probability characteristics should be considered. Besides, the complex relationships between the parameters of the modeling system are not linear; their linearization leads to a discrepancy between the object and the model, therefore the results obtained are not able to accurately express the real patterns in advance. It is therefore necessary to develop such methods that will take into account the probability nature of the variables in them and their actual dependencies. As a rule, in complex systems the decision is made in conditions of partial or complete uncertainty, therefore the expected results cannot also be completely determined, i.e. their receipt is associated with a certain risk, the existence of which depends on the probability nature of externalities, as well as the incomplete determination of the processes in the current system. In the professional literature known to us, there are only vague approaches or hints about getting possible credit risk assessments. In some ways it is recommended to use the average value of expected profit (or mathematical expectation) or the expected profit dispersion (or average squared deviation from the mean) as criteria for quantitative risk assessment [Sevruk, 2001].

We think that these assessments can not fully represent the reality, as they provide little information about the financial and credit policy of banks. Thus, it is necessary to develop a new probabilistic approach to measuring the risk involved in decision making. Below we present a generalized method of solving the above-mentioned problem, which can have both practical and versatile application.

1. Mathematical analysis of statistical data on the amount of loans provided by commercial banks to enterprises and households shows that the figure of loans, as random variables, are subject to the exponential law of probability distribution [Sevruk 2001].

$$F(x)=1-\boldsymbol{e}^{-\frac{x}{T}} \qquad (1.1)$$

where x is a random variable expressing the amount of provided loans, T is the average loan amount, which will be called the bank's loan constant. The differential law of probability distribution (or the probability density function) will be

$$\mathbf{f(x)} = \frac{\mathbf{1}}{\mathbf{T}} * \mathbf{e}^{-\frac{\mathbf{x}}{\mathbf{T}}} \qquad (1.2)$$

The compliance of this law and statistic data was verified by Pearson χ^2criterion and a high probability of their compliance is obtained (P>0.9):

So, based on the probability distribution function of the loans and the probability density function (PDF), the following approach can be proposed for risk assessment.

2. Let's consider banks' credit policy formalization as a transformation of x-amount credit, which turns it into Q profit in the current economic climate and is also considered a random variable. Meanwhile, it can be admitted with great accuracy that the amount of profit received due to financial and credit activities of the bank is directly proportional to the amount of the loan [Sevruk, 1996, 72], i.e.

$$\mathbf{Q}(\mathbf{x}) = \begin{cases} \mathbf{0}, \ \mathbf{x} > \mathbf{0} \\ \mathbf{k} * \mathbf{x}, \ \mathbf{0} < \mathbf{x} < \boldsymbol{x_{max}} \\ \mathbf{0}, \ \mathbf{x} > \boldsymbol{x_{max}} \end{cases} \qquad (1.3)$$

where $\boldsymbol{x_{max}}$ is the largest possible loan amount provided by a bank, $\boldsymbol{k}$ is the ave-rage annual interest rate on loans. The formalized scheme of profit generation is as follows (Figure 1).

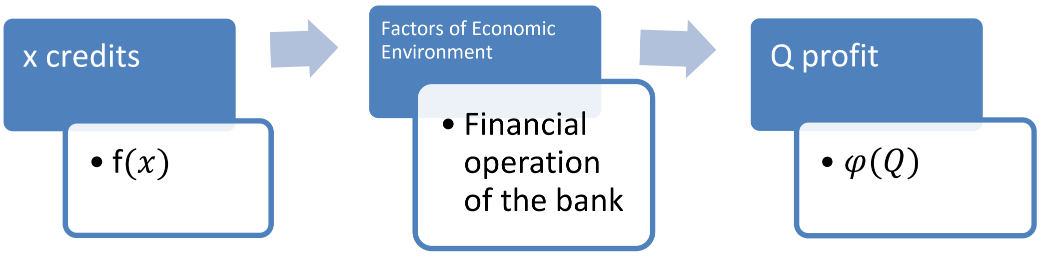

Figure 1. Profit generation scheme

Let's assume that the variety of possible solutions is a positive, continuous random variable with a probability density function (PDF) f (x). Suppose that the decision x makes a result of Q (x), for example, profit. Generally speaking, Q (x) result indicator is also a continuous random variable with a function φ (Q) of its hitherto unknown probability distribution density. For a particular system under observation, it is always possible to determine the pattern of Q (x) output obtained as a result of the decision made. To carry out quantitative risk assessment within the decision made, we propose to calculate the probability of the occurrence according to which Q of the expected outcome would be smaller than the desired Q_0 level given in advance, i.e. $\mathbf{P}(\mathbf{Q} < \boldsymbol{Q_0})$, or Q variable would be in a certain undesirable range $(\mathbf{Q_1}, \mathbf{Q_2})$, i.e. $\mathbf{P}(\mathbf{Q_1} < \boldsymbol{Q} < \boldsymbol{Q_2})$.
Thus, for quantitative risk assessment, the following formulas are obtained in the first and second cases respectively:

$$\mathbf{R} = \int_{\mathbf{0}}^{\mathbf{Q_0}} \boldsymbol{\varphi}(\mathbf{Q})\mathbf{dQ}$$

(1.4)

$$\mathbf{R} = \int_{\mathbf{Q_1}}^{\mathbf{Q_2}} \boldsymbol{\varphi}(\mathbf{Q})\mathbf{dQ}$$

To apply the formulas in practice, it is necessary to find the function of the random distribution density Q (x), which in turn depends on the random quantity x of the probability distribution variable f (x).

As we have already observed [Ekonomicheskaya bezopasnost, 2001], the nature of the law f (x) does not change in the case of a linear change in the *x* variable, so the function φ (Q) will be exponential.

The solution to this problem is already known and, by following example [9], the function φ (Q) would be:

$$\boldsymbol{\varphi}(\mathbf{Q}) = |\mathbf{f}[\boldsymbol{\psi}(\mathbf{Q})]| * \boldsymbol{\psi}'(\mathbf{Q}) \qquad (2)$$

where ψ (Q) is the inverse of the Q (x) function of obtaining the output index.

Let's observe the quantitative risk decision by the example of the bank's credit risk, considering that it pursues a fairly cautious credit policy, which is one of three possible strategies:

fairly cautious

moderately cautious

relatively courageous

As it has already been mentioned, research [Venttsel, 1974] shows that in case of a fairly cautious credit policy, the amount of loans provided has a probability density distribution function.

$$\mathbf{f}(\mathbf{x}) = \frac{\mathbf{1}}{\mathbf{T}} * \mathbf{e}^{-\frac{\mathbf{x}}{\mathbf{T}}} \quad , x>0 \qquad (3)$$

where *T* is the average loan amount variable.

As it has already been mentioned, in the case of a simple, linear representation of the output Q (x), which is typical of bank's loan operations, the following occurs:

Q(x)=k*x (4)

where *k* is the average interest rate on the loan.

From (3) we can get the inverse function ψ (Q) and its derivative.

$$\boldsymbol{\psi}(\mathbf{Q}) = \frac{\mathbf{x}}{\mathbf{k}} \ , \qquad \boldsymbol{\psi}'(\mathbf{Q}) = \frac{\mathbf{1}}{\mathbf{k}} \qquad (5)$$

Taking into account (2) and (5), the function of density distribution **φ (Q)** can be obtained in the case of cautious credit policy.

$$\boldsymbol{\varphi}(\mathbf{Q}) = \frac{\mathbf{1}}{\mathbf{T*k}} * \mathbf{e}^{-\frac{\mathbf{Q}}{\mathbf{T*k}}} \qquad (6)$$

Via formula (6) we get the probability or risk that the result index **Q (x)** will be below a certain desired level of **Q_0** given in advance, as we defined it as a quantitative-probability risk assessment of the decision made.

$$\mathbf{R = 1 - e^{-\frac{Q_0}{T*k}}} \quad (7)$$

Thus, the variable of the profit is subject to the exponential law of probability distribution, i.e. $\mathbf{M(Q) = k * T}$ by mathematical expectation.

As a final result, we can assess the risk by the expected profit of mathematical expectation and dispersion, using formula (7) and calculating the dispersion by the following final expression (8).

$$\mathbf{D(Q)=(\mathit{T}-1)^2 \approx \mathit{T}^2} \qquad \mathbf{\sigma(Q)=\sqrt{D(Q} \approx \mathit{T}} \quad (8)$$

For quantitative risk assessment, according to law (6), a number of problems of theoretical and practical value can be solved, for example, to determine:

- $P(Q > Q_0)$ and $P(Q < Q_0)$ probabilities
- $P(Q < T)$ probability of receiving less than average profit
- the average interest rate at which $Q=Q_0$ in case of R risk
- $P(Q_1 < Q < Q_2)$ probability, etc.

Conclusion. The approach to quantitative assessment of credit risk discussed in this article is interesting in respect that in case of different functions of crediting vo-lumes and expected profit, it would be possible to calculate the credit risk and its sensiti-vity to changes in various parameters according to the strategy chosen by the bank.

The scientific novelty of the work is summarized in the proposed model, which al-lows to assess the credit risk based on the probability distribution density function of the provided loans, according to the different scenarios of the strategies chosen by the bank.

The end results of the work are conditioned by the fact that credit volumes are ran-dom variables of exponential distribution, but the above discussed approaches can also be applied in case of arbitrary distribution of credit volumes and to obtain formulas for calculating risk or other equivalent indicators.

In a nutshell, the presented approach can be used to quantify the risk in different situations and fields.

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Narek KESOYAN, Mesrop MESROPYAN

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Currently, various mathematical methods and models are used for quantitative assessment for risks, most of which are linear, as in case of nonlinear models normal distribution of random variables is assumed. This approach, in our opinion, significantly devalues the model, since other distributions of random variables that change over time are more common. It is known that the financial and credit policy pursued by banks in connection with large investments is carried out in conditions of certain risks. As a result, we come to a situation where there is some degree of uncertainty, when the expected return on bank investment in a loan portfolio is a random variable or random function. In such conditions, the development of new methods of quantitative analysis and assessment of the risk contained in the financial operations of a bank is becoming a very urgent problem, the solution of which will allow financial institutions to estimate in advance the likelihood of obtaining the expected profit, or calculate estimates of other equivalent indicators. In this article tools of probability theory were used as well as modern risk classifying systems and management strategies. Thus, the presented approach can be used in decision making in different conditions and fields for quantitative risk assessment.