SUBDIVISION OF THE OBJECT INTO SOLID POLYGONS AND ITS APPLIED AND ECONOMIC ASPECTS

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Introduction. Algorithms for dividing an object into convex polygons are among the main tools in computer graphics, robotics, and machine vision. These algorithms make it possible to divide complex objects into simpler subsets consisting of convex poly-gons, which makes possible to simplify their processing, analysis and visualization.

Convex polygons have many advantages over more complex objects. First, they are easily described mathematically and can easily be stored in computer memory. Second, they have simple geometric properties, which allows you to quickly determine their position in space. Third, convex polygons are easily sorted and processed using standard computer graphics algorithms.

The method described in this article is used in measuring the distance between objects. By dividing the object into convex components, the distance between the objects can be measured using the Gilbert-Johnson-Kirth (GJK)[1] algorithm.

Methodology. In this article, an algorithm has been developed that divides the object into convex polygons in the most optimal way possible. Having 3 points, the convexity of an angle is checked using trigonometric functions. To determine the points of intersection of the cuts, a system of equations is built, the elements of which are the sides of the object and the cuts. We solve the system of equations using the Gauss method.

Literature review. The process of triangulation can be used to fragment a complex object into a set of convex polygons. This involves breaking the object down into a series of triangles that form a mesh of convex polygons, which can be useful for various applications in computer graphics and computational geometry [Mark de Berg, Over-mars, 2008, 45-60]. Ear-clipping is a popular method for dividing a non-convex polygon into convex polygons by iteratively removing "ears" from the polygon until only convex polygons remain. The method works by finding a triangular "ear" and removing it, and then iterating until the entire polygon has been divided into convex parts[Mark de Berg, 2008, 55-59]. Constrained Delaunay triangulation is a method of triangulating a set of points in a plane, subject to constraints such as edges or boundaries. The resulting triangulation conforms to the constraints and satisfies the Delaunay criterion for optimal triangle quality[Edelsbrunner, Shah, 1996, 223-241].

Scientific novelty. At present, the demands of the manufacturing market are increasing and with it, the demand for software is increasing.Our developed algorithm enables efficient partitioning of the object into convex polygons. In contrast to the above-mentioned algorithms, the algorithm developed in the article divides the object with as few reductions as possible, which is of great importance in the areas mentioned in the article.

Analysis. Algorithms for dividing an object into convex polygons can be classified into two types: cut-based algorithms and set-partitioning algorithms.

Cut-based algorithms work by constructing cuts (planes) that pass through an object and divide the object into two or more parts. This process is repeated until convex polygons are obtained.

Polygon-based algorithms use a recursive process to divide an object into simpler subsets using its geometric properties. These algorithms can be more efficient than slicing algorithms, but they can be more difficult to implement.

A non-convex polygon is a polygon that has at least one interior angle greater than 180 degrees. The number of possible vertices that a non-convex polygon can have is at least four since any polygon with three vertices is necessarily convex.

In a non-convex polygon, the number of possible vertices greater than 180 degrees depends on the number of vertices. If a non-convex polygon has n vertices, then the maximum number of interior angles that can be greater than 180 degrees is n-2. This is because the sum of the interior angles of a polygon with n vertices is (n-2) * 180 degrees and every angle in a convex polygon must be less than 180 degrees.

For example, in a pentagon (a 5-sided polygon), the maximum number of interior angles that can be greater than 180 degrees is 3 (n-2=5-2=3).

This formula can be proved mathematically by using the fact that the sum of the interior angles of an n-sided polygon is (n-2) * 180 degrees.

Let x be the number of interior angles greater than 180 degrees in a non-convex polygon with n vertices. Each interior angle greater than 180 degrees contributes an additional 180 degrees to the sum of the interior angles. Therefore, the sum of the interior angles of a non-convex polygon is as follows (1):

$$(n-2) * 180 + x * 180 = (n+x-2) * 180 \quad (1)$$

But since the sum of internal angles of the polygon is (n-2) x 180, then it will be:

$$(n + x - 2) * 180 = (n - 2) * 180 \quad (2)$$

Simplifying this equation (2), we get:

$$n + x - 2 = n - 2$$

$x = 0$

This means that a non-convex polygon cannot have interior angles greater than 180 degrees, which is a contradiction. Therefore, it turns out:

$$n + x - 2 > n - 2$$

$x > 0$

Thus, in a non-convex polygon with n vertices, the maximum number of interior angles greater than 180 degrees is n-2.

That is, in the best case, a non-convex polygon shares one line and intersects two convex polygons.

The question of how to divide a non-convex polygon into convex polygons is the main topic of this article.

Suppose we have the polygon ABCDEFGH (Fig. 1).

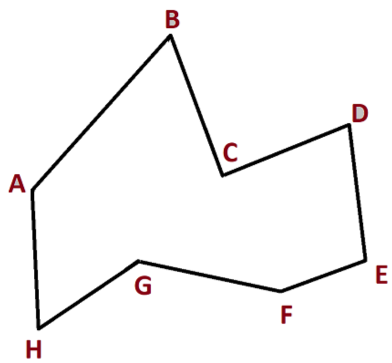

Figure 1. Non-convex polygon

The developed algorithm iteratively moves through the vertices of the polygon, checking the interior angles of the polygon. If the angle is greater than 180°, then the side formed by that vertex and the preceding vertex continues until it intersects any side of the polygon.

Reaching vertex C, the side BC continues until it intersects the polygon at point j and divides the polygon into two polygons (Fig. 2).

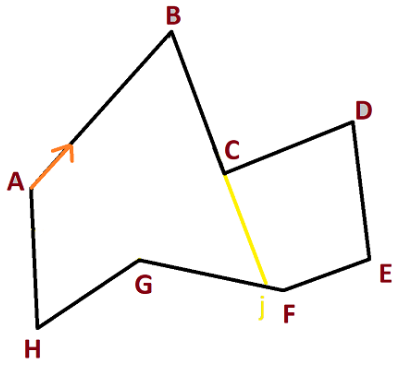

Figure 2. Direction and intersection of motion with polygon vertices

The algorithm iterates through the vertices until it again encounters an angle greater than 180°. Fragments the object again, checking the intersection of the path with the polygon and the segment Cj (Fig. 3).

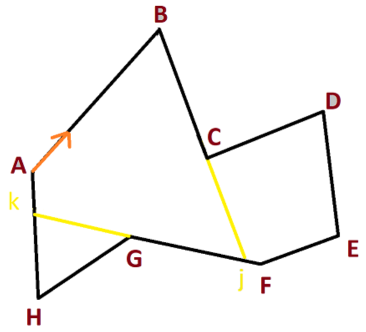

Figure 3. The final view of the split polygon

In the next example, we consider a case on the polygon ABCDEFG, when the dividing lines intersect. The movement starts from vertex A (Fig. 4).

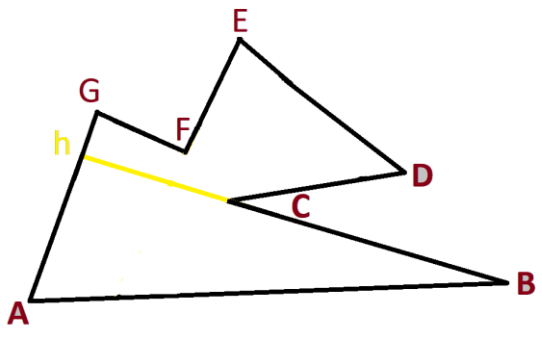

Figure 4. The first intersection of the polygon

Reaching the vertex C, the polygon splits into two parts along the direction Ch. Each line through which a polygon is intersected is stored in memory for later determination of intersection points. Then it continues to move to the next angle greater than 180°, which is the angle adjacent to vertex F in the above polygon. With the above method, it continues the line EF until it intersects with the line, which in our example is the line Ch (intersections with cuts are checked in advance). The intersection point is designated by the letter i (Fig. 5).

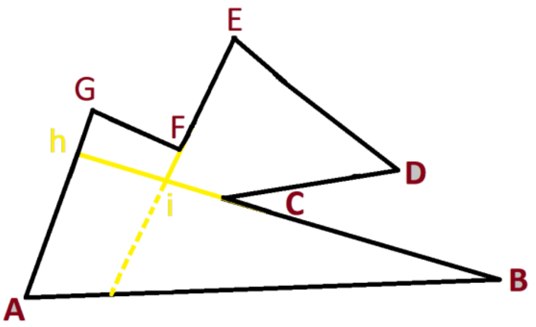

Figure 5. Intersection of lines intersecting a polygon.

The algorithm continues to move through the vertices of the polygon until all vertices have been considered. As a result, having a heptagonal non-convex polygon ABCDEFG, 3 convex polygons are obtained.

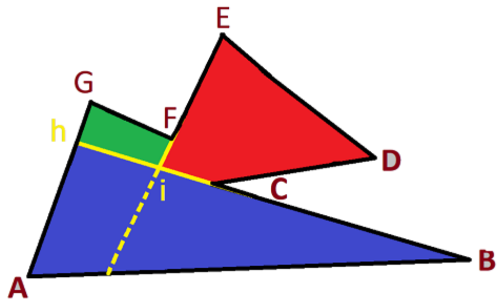

Figure 6. The final view of the broken polygon after the intersecting lines have been *crossed.*

It is not difficult to notice that if the number of angles of a polygon greater than 180° is n, then because of division, an n+1 convex polygon is obtained.

If the vertices of the polygon are given, the equations of the sides of the polygon are created to determine the point of intersection of the lines. And if the equation of the lines of the polygon is given, then a system of equations is immediately created, which, by solving, gives the coordinate of the point of intersection.

To reduce the number of equations, the vertices adjacent to the vertex can be excluded. The resulting equations can be solved by the Gauss method [Gilbert Strang, 2016, ch11].

Comparing the algorithm developed in this article with triangulation [Mark de Berg, Overmars, 2008, 45-60] from the well-known algorithms, it can be said that this algorithm divides the object into convex polygons by dividing the object as little as possible. The disadvantage of the algorithm is that after each intersection, the resulting line is compared with all the other edges, except for the edges adjacent to the vertex.

Dividing an object into convex polygons has many practical and economic aspects that can be useful in various fields, including computer graphics, architecture, manufacturing, and many others fields. Some of these aspects may include:

Calculation of volumes and areas. when dividing an object into convex polygons, the volumes and areas of its parts can be calculated. This can be especially important in construction and manufacturing, where accurate calculations of volumes and areas can help reduce material costs and optimize production processes.

Modeling. Dividing an object into convex polygons can help create a 3D model of the object. This can be useful in computer graphics and architecture, where 3D models can be used for visualization and design.

Error detection. When dividing an object into convex polygons, it is possible to detect errors and inconsistencies in its design. This can be especially important in manufacturing, where even the smallest mistakes can lead to serious problems.

Cost calculation. When dividing an object into convex polygons, you can calculate its value. This can be especially important in manufacturing, where accurate costing can help reduce material costs and optimize production processes.

Process simulation. Dividing an object into convex polygons can help model the processes associated with the object. For example, this can be used to simulate the assembly process or to test an object before production.

Data management. By dividing an object into convex polygons, you can more easily manage the data associated with the object. For example, this can be used to create a database containing information about every part of an object. Additionally, dividing the object into convex polygons can help reduce the amount of computing resources required to process the object. This is especially important for manufacturing and engineering applications, where calculations and analysis can be time-consuming and require significant computing power. Dividing an object into simpler, more convex polygons can help reduce the number of calculations and reduce system overhead.

Conclusion. The described algorithm moves along the vertices of a non-convex object and checks the interior angle. If the internal angle is greater than 180°, we divide the object by the continuation of the edge formed by the given vertex and the vertex preceding it. During each cut, we study the presence of intersection points with other cuts

Thus, dividing an object into convex polygons has many applications and can help reduce manufacturing costs, improve model quality, and optimize manufacturing processes. And with the algorithm discussed in this article, object fragmentation is done in the most optimal way possible.

References

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Subdivision of the object into convex polygons and its applied and economic aspects

Keywords: polygon division, subset, concave polygon, the intersection of lines, vertex, movement with vertices

In this article, we have studied an efficient algorithm by which we can efficiently divide a geometric object into convex components.Algorithms for dividing an object into convex polygons are among the main tools in computer graphics, robotics, and machine vision. These algorithms divide complex objects into simpler subsets consisting of convex polygons, which makes it possible to simplify their processing, analysis and visualization.Convex polygons have many advantages over more complex objects. First, they are easily described mathematically and can easily be stored in computer memory. Second, they have simple two-dimensional properties, which allows us to quickly determine their position in space. Third, convex polygons are easily sorted and processed using standard computer graphics algorithms.The described algorithm moves along all the internal angles of the object and checks whether that internal angle is greater than 180°, if so, divides the object by the continuation of the side forming that vertex. The advantage of our developed algorithm is the optimal partitioning of the object by dividing it into as few convex polygons as possible. And the shortcoming of the algorithm is its complexity.