OPTIMAL ALLOCATION OF PENSION FUNDS AND RISK BUDGETING

Karine AVETISYAN
Ph.D. in Economics, Associate Professor
Enterprise Incubator Foundation Project Manager
Rita HOVHANNISAN
Ph.D. Student at YSU
Armbususinessbank CJSC, Head of CTPR division

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Introduction. In order to achieve maximum efficiency of the pension fund investments, the latter should be optimally allocated based on the risk diversification principle. The main goal of pension fund managers is to manage the portfolio under various asset investment restrictions and asset allocation strategies under a well-defined optimality criterion. An optimal portfolio has been built under various restrictions on the investment of pension fund assets. Greater attention is paid to the optimum portfolio risk diversification measures such as marginal contribution to risk (MCTR), asset contribution to risk (ACTR) and percentage contribution to risk (PCTR), which are used to represent the relationship between asset weight and portfolio risk.

Methodology. Theoretical basis of the research are the classic articles on mean-variance optimization, risk budgeting and diversification. The studied literature gives a comprehensive and multi-level view of the role of the relationship between asset weights and portfolio risk. The empirical part of the analysis is based on data available from RA Central Bank, Bloomberg and Yahoo Finance. Optimal portfolio analysis is done in MATLAB based on linear boundary conditions for asset, currency and country groups.

Literature review. The total risk of a portfolio depends on the risk of the assets included in it, the magnitude and sign of their correlation with the portfolio, the nature of the weight of each asset in the portfolio. The weight of the asset in the portfolio does not imply the same share in the risk of the portfolio. The share of an individual asset in the risk of the portfolio depends on the risk of the asset, its weight in the total portfolio, and the correlation between the returns on the asset and the portfolio. Diversification is a way to reduce risk by directing investments to assets which will respond to the same event differently. The main goal of diversification is to reduce risk. Marginal contribution to risk (MCTR), asset contribution to risk (ACTR) and percentage contribution to risk (PCTR) are considered as indicators of portfolio diversification level [Thierry Roncelli, 2020, 104-117]. The latter are used to represent the relationship between asset weight and portfolio risk. There are different approaches regarding optimal asset management. The most common and classic model, which is the basis of many other models, is the Markowitz model [Kasilingam, 2014, 187-204]. According to Markowitz, in the

optimal portfolio selection the return of the portfolio and its risk shall be considered together, where the risk is measured by standard deviation of asset returns. Markowitz' classic model is used as a basis for evaluating the optimal portfolio. The setup of the model is the Markowitz model under various both equality and inequality constraints:

$$\Pi(w) = w^T \times \mu - \lambda \times w^T \times \Sigma \times w,$$
$$A \times w = b,$$
$$C \times w \leq d,$$
$$\mu_p(w) = w^T \times \mu, \qquad \sigma_p(w) = (w^T \times \Sigma \times w)^{0.5},$$

where μ is a $Nx1$ vector, whose i element is the expected return of the i^{th} asset; ω is an $Nx1$ vector whose i element is the weight of the corresponding asset in the portfolio, $\Sigma\omega = 1$; μ_p and σ_p are the return and standard deviation of the portfolio, respecttively; Σ is a covariance matrix of NxN asset returns, λ is the risk tolerance coefficient.

Risk budgeting allows to assess the extent to which assets have an impact on portfolio risk or which assets diversify portfolio risk. For risk budgeting, portfolio's annualized VaR is considered at 99% confidence level. VaR of portfolio is assessed assuming that asset returns are jointly normally distributed [Roncelli, 2020, 69-88].

$$VaR_{p,Student}(w) = -\left(\mu_p(w) + \sigma_p(w) \times Z_{normal,1-\alpha}\right),$$

where $Z_{normal,1-\alpha}$ is the quantile function of univariate standard normal distribution and $(1-\alpha)$ is the confidence level.

MCTR, marginal contribution to risk, shows the effect of changes in the weight of each asset on the portfolio risk. It has the following mathematical representation:

$$MCTR = \left(\frac{\delta VaR_p(w)}{\delta w_1}, \dots, \frac{\delta VaR_p(w)}{\delta w_N}\right)^T = \frac{\delta VaR_p}{\delta w} = -\left(\mu + \frac{\Sigma \times w}{\sigma_p} \times Z_{1-\alpha}\right),$$

ACTR, asset contribution to risk, indicates the contribution of each asset to the portfolio risk and its mathematical representation is:

$$ACTR = diag(w) \times MCTR = -diag(w) \times \left(\mu + \frac{\Sigma \times w}{\sigma_p} \times Z_{1-\alpha}\right)$$

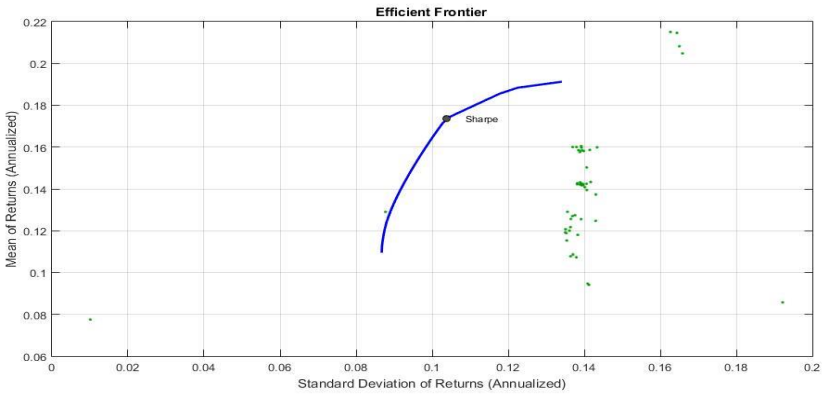
PCTR, percentage contribution to risk: share of asset's risk in risk of portfolio:

$$PCTR = \frac{diag(w) \times MCTR}{VaR_p} = -\frac{diag(w) \times \left(\mu + \frac{\Sigma \times w}{\sigma_p} \times Z_{1-\alpha}\right)}{VaR_p}:$$

Scientific novelty. The scientific novelty of the research is the analysis of eligible asset classes for pension funds under looser boundary conditions than imposed limits by law based on the mean-variance portfolio optimization model where marginal contribution to risk (MCTR), asset contribution to risk (ACTR) and percentage contribution to risk (PCTR) are used as diversification measures to describe the relationship between asset optimal weights and portfolio risk. The proposed analysis can be used in optimal management of pension funds for asset allocation purposes.

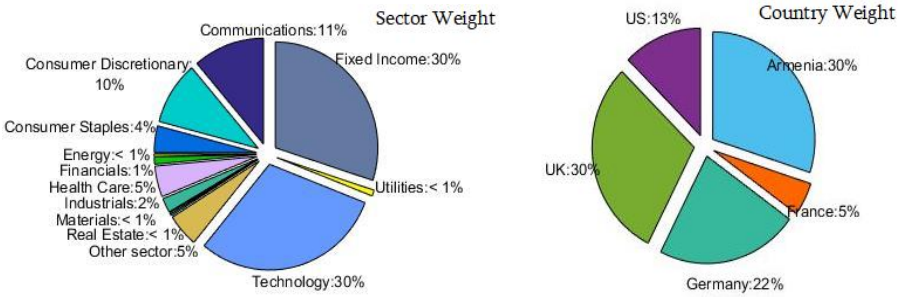
Analysis. Optimal portfolio assessment and risk budgeting is done in MATLAB program. An optimal portfolio is built under different restrictions. The optimal portfolio is defined to be the portfolio corresponding to Sharpe's maximum coefficient [Alexander et al., 2015, 111-135]. For the construction of the portfolio, the unhedged returns are first calculated based on the return of assets in Armenian drams. Optimal portfolios are constructed using restrictions based on the currency of the asset in which the trade is conducted as well as restrictions on country limits. As there are missing data in the asset price time series, the expected return and covariance matrix are estimated by the missing data method, and then the covariance matrix is filtered based on the random matrix theory. For the optimal portfolio, risk statistics are assessed by currency, country and sector. In order to evaluate the optimal portfolios, investments in ETFs (Exchange traded fund) and RA government bond indices / ETF and index, currently active / are considered. Data are monthly for the period ranging from 01.01.2014 to 28.02.2022, Bloomberg. The selection of ETFs is done according to the following criteria: monthly price, Morningstar Performance Rating 4 and 5 stars, region: USA, Great Britain, France, Germany, market capitalization of more than 100,000, 3-month average volume of more than 10,000, turnover coefficient is greater than 1 /the source is https://finance.yahoo.com/etfs/. As a result, 48 ETFs are selected with the above parameters. The indices of treasury securities with up to one-year remaining time to maturity (TBI) and more than 5 years of remainning time to maturity (G5I) are considered for AMD investments (www.cba.am}. Restrictions by country and currency are defined as follows: as a lower limit - a minimum of 5% in each country, and in the case of a currency - 10%; the return on risk-free assets in this paper is estimated at 8%. As a result of the optimization, based on the expected return-risk assessments for the above assets, the efficient frontier is calculated, Chart 1. The diagram also shows Sharpe's optimal portfolio.

Chart 1. Effective limit. Sharpe coefficient: 0.26



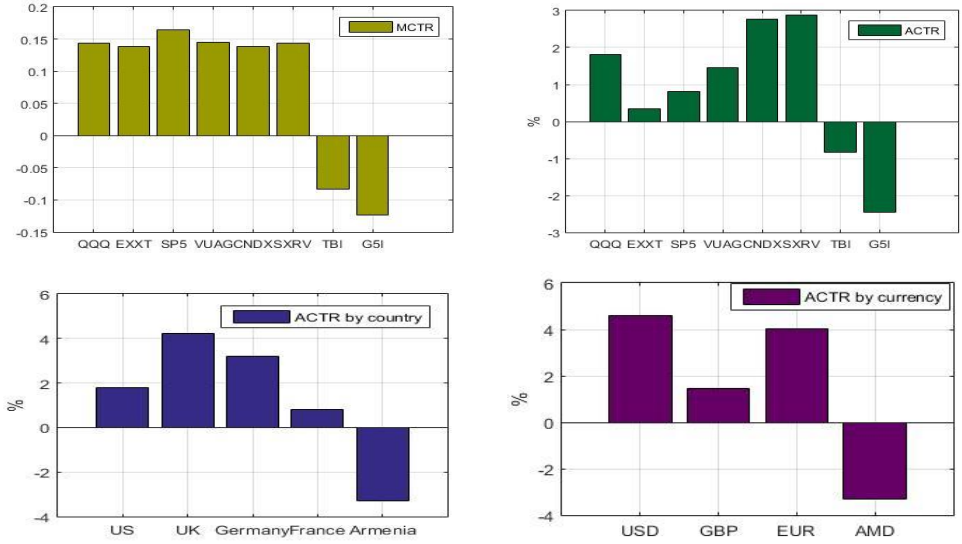
The graphs in Chart 2 show the optimal weight distribution of the Sharpe portfolio by sector and country.

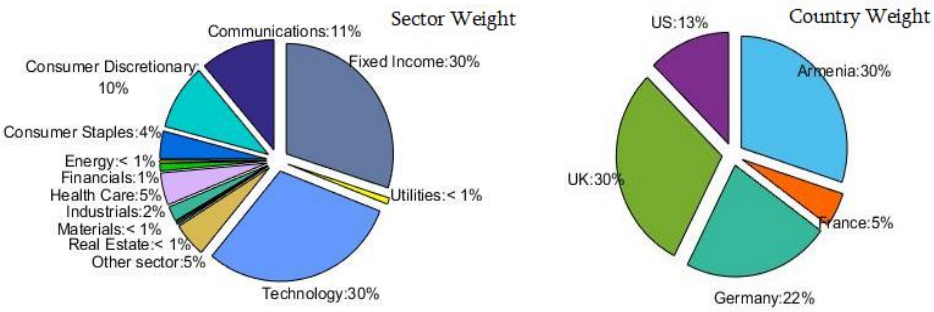
Chart 2. Distribution of Sharpe portfolio weights by country and by sector



Next, graphs in Chart 3 show the MCTR and ACTR by country and currency.

Chart 3. MCTR ACTR by country and currency





Next, graphs in Chart 4 show the MCTR and ACTR by country and currency.

Chart 4. MCTR ACTR by country and currency

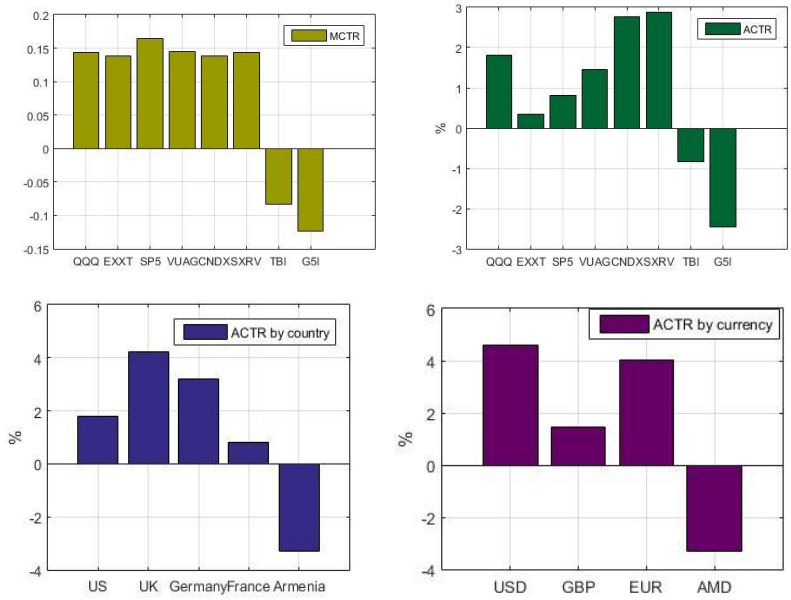
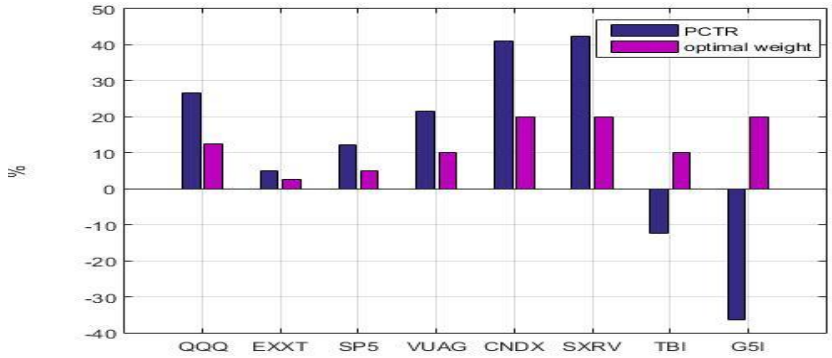


Chart 4 also shows the asset sensitivity to portfolio risk. For example, a change in the weight of the QQQ index in the MCTR chart results in a 0.14-point increase in portfolio risk. Though, in the optimal portfolio TBI and G5I together make up 30% of the portfolio, they play a role of diversifying assets, reducing the risk of the portfolio. The ACTR diagram shows how much risk comes from each index and shows the risk distribution of the portfolio. In ACTR by country and currency chart the risk is the share of assets by currency and country. 4.58% of the 6.77% risk of the portfolio comes from USD, 1.45% from GBP, 4.03% from EUR, and -3.29% from AMD, moreover: 1.8% of the risk comes from the USA, 4.23% comes from Great Britain, 3.21% from Germany, 0.82% from France, and -3.29% comes from Armenia. Next, in Chart 4, the share of each asset's risk in the optimal portfolio risk is calculated.

Chart 5 shows that TBI makes up 10% of the optimal portfolio, but the latter plays a risk-reducing role. In parti-cular, the weight of TBI risk in the total portfolio is -12-26%. The same applies to the G5I index. As a result, optimal portfolio management can have a significant impact on portfolio risk diversification. Depending on the share of assets in the portfolio, the de-pendence between the asset and the portfolio, as well as the risk of the asset, the participation of a given asset in the total risk of the portfolio is different.

Chart 5. Percentage of PCTR risk, optimal weights



Conclusions. The optimal portfolio is built under the restrictions, and portfolio corresponding to Sharpe's maximum coefficient is considered as the optimal portfolio. Then, a study of the diversification level of the portfolio is carried out by using the marginal contribution to risk (MCTR), asset contribution to risk (ACTR) and percentage contribution to risk (PCTR) as measures. As a result of the analysis, it turned out that the AMD assets play a role of diversifying assets in the portfolio. In particular, in contrast to foreign currency assets, the risk-weighted participation of AMD assets is negative, which leads to a reduction in the overall risk of the portfolio.

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An optimal portfolio is built under various restrictions on the investment of pension fund assets. Greater attention is paid to the level of optimal portfolio diversification: sensitivity of the risk measure to asset weights and assets' correlation with the portfolio are used to represent the relationship between asset weight and portfolio risk. The Markowitz model setup with equality and inequality constraints is used to evaluate the optimal portfolio. Optimal portfolio assessment and risk budgeting are estimated in MATLAB. The optimal portfolio is considered to be the one which has the highest Sharpe ratio. As a result of the optimization, the efficient frontier and Sharpe ratio are estimated, also the optimal weight distribution of Sharpe portfolio by sector and country is calculated. Then, a study of the diversification level of the portfolio is carried out, by using the marginal contribution to risk (MCTR), asset contribution to risk (ACTR) and percentage contribution to risk (PCTR) as diversification measures.