THE ANALYSIS OF FIRST PRICE SINGLE BID AUCTION WITH UNIFORMLY DISTRIBUTED TYPES WITHIN THE SCOPE OF MECHANISM DESIGN THEORY

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Introduction. Mechanism design theory has widespread applications across various fields of economics. The exploration of mechanism design and its market implementations has attracted considerable attention from researchers. The primary aim of mechanism design theory is to develop mechanisms that achieve pre-defined objectives. Mechanism design operates within the constraints of informational asymmetry and individual rationality. In this context, agents act in their self-interest, striving to maximize their utility. Meanwhile, the mechanism designer aims to achieve desirable social and economic outcomes for all parties, guided by specific criteria of interest. Previous studies by the authors have offered the particular solution of the first price sealed bid auction consisting of two agents. The goal of this article is to extend this ana-lysis to the first price sealed bid auction consisting of N agents, employing mechanism design theory as the analytical framework. To achieve the defined goal the following objectives will be addressed:

- analyze the first price sealed bid auction consisting of N agent using the methodology and tools provided by mechanism design theory,
- analze the first price sealed bid auction consisting of N agents employing simulation tools to understand its dynamics,
- identify Bayesian dominant strategies for the agents and determine Bayesian Nash equilibrium of the auction,
- discover social choice function that is implemented by the mechanism of first price sealed bid auction,
- assess the probability of agents winning when they apply Bayesian dominant strategies, considering the type of agent,
- calculate expected utilities of agents and the expected revenue of the seller, providing insights into the auction’s efficiency,
- evaluate the impact of increasing the number of agents on various auction performance metrics

The object of the article is the first price sealed bid auction consisting of N agents, and the subject of the article are the problems of design and implementation of the first price sealed bid auction.

Literature review. In recent decades, mechanism design theory has captivated the economic community, boasting both theoretical and practical applications. This theory finds wide-ranging applicability in various markets, organizational processes, and in the design of new auction formats, contracts, jurisdictions, among other areas [Royal Swedish Academy, 2007, 1-19; Jackson, 2014, 1-8]. Particularly notable is the theory's efficiency in auction theory. Mechanism design theory facilitates the analysis and comparison of different auction types, assessing the efficiency of their implementation and application. Leveraging this theory, new auction models have been developed, whose equilibrium states align with the mechanism designer's initial goal function [Milgrom, 2004, 1-62].

Methodology. The article conducts a detailed analysis of the first price sealed-bid auction. In such auctions, all participants submit their bids simultaneously without knowledge of the others' bids. The item is awarded to the participant who submits the highest bid, and this winning bidder pays the price they proposed [Lebrun, 1996, 421-443], [Despotakis, et al., 2021, 888-907]. This analysis specifically focuses on auctions where a single, indivisible item is sold. Within the domain of mechanism design theory, the term θ_i is utilized to represent the type of an agent. This framework facilitates agents in making collective decisions. Prior to engaging in the decision-making process, each agent privately observes a distinct parameter or message, which delineates his preferences and, consequently, influences his utility function. Mathematically, this concept is articulated by incorporating the parameterΘ_i, which is exclusively observed by agent.

In the utility function $u_i(a, \theta_i)$, the inclusion of the θ_i parameter signifies that an agent's type directly affects his preferences and utility function. Specifically, in the context of a first-price sealed-bid auction, the type θ_i indicates an agent's valuation or willingness to pay for the product, whereas Θ_i denotes the set of all possible types [Colell, et al., 1995, 857-897].

$f: \Theta_1 \times \Theta_2 \times \ldots \times \Theta_N \rightarrow X$ social choice function defines an alternative $f(\theta) = x \in X$ for each type vector $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_N)$. Social choice function is ex post efficient or Pareto efficient, if for any type vector $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_N)$ there is no alternative $x \in X$, where $u_i(x, \theta_i) \geq u_i(f(\theta), \theta_i)$ for each agent i, and $u_i(x, \theta_i) > u_i(f(\theta), \theta_i)$ for some agent. The social choice function of the first price sealed bid auction is Pareto efficient, if the product is sold to the agent with the highest valuation.

Mechanism $\Gamma = (S_1, S_2, \ldots, S_N, g(\cdot))$ is a set of N strategy vectors and $g: S_1 \times S_2 \times \cdots \times S_N \rightarrow X$ result function. Each agent observes his θ_i type and based on some S_i strategy sends a message to the mechanism, which makes a collective decision based on $g(\cdot)$

result function and chooses an alternative x from the set of X alternatives. In first price sealed bid auction all the agents privately observe their types and place bids based on a strategy. According to the result function of the mechanism the product is sold to the agent, who placed the highest bid ($y_i(b) = 1\ if\ b_i = \max\{b_1, b_2, \cdots, b_N\}, 0\ otherwise$), where b_i is the bid of the agent i and b is the vector of all bids. For the auction the alternative also consists of t_i payments, which align the result function of the mechanism with the highest bid $t_i = -b_i \times y_i(b)$. The description of the first price sealed bid auction as a mechanism is given bellow:

(1) $\Gamma = \left(S_1, S_2, \dots, S_N, g(\cdot)\right)$

(2) $s_i \in S_i \in R_+$

(3) $g(b) = x \in X = \begin{bmatrix} y_i(b) = 1\ \text{եթե}\ b_i = \max\{b_1, b_2, \cdots, b_N\} \\ y_i(b) = 0\ \text{եթե}\ b_i \neq \max\{b_1, b_2, \cdots, b_N\} \\ t_i = -b_i \times y_i(b) \end{bmatrix}$

$\Gamma = \left(S_1, S_2, \dots, S_N, g(\cdot)\right)$ mechanism implements the social choice function, if there exists a vector of strategies $(s_1^*, s_2^*, \dots, s_N^*)$, which results in an equilibrium state (Nash equilibrium, Bayesian Nash equilibrium, etc.) for the Γ mechanism [Maschler, et al.,, 2013, 75-313], where the result function and the social choice function are equal $(s_1^*(\theta_1), s_2^*(\theta_2), \dots, s_N^*(\theta_N)) = f(\theta_1, \theta_2, \theta_3, \dots, \theta_N)$.

Γ mechanism is called direct mechanism, when the sets of types and the strategy sets of the agents coincide, and the social choice function is consistent with the result function. Particularly $S_i = \Theta_i$ and $g(s(\theta)) = f(\theta)$. In direct mechanisms the agents directly reveal information regarding their types. While in indirect mechanisms the agents reveal information based on their θ types and S strategies, which is sufficiently different from the types. The first price sealed bid auction is considered as a direct mechanism, because the agents reveal information regarding their types or valuations. The social choice function which is implemented by the first price sealed bid auction is provided in the analysis part of the article. The social choice function f is truthfully implementable (incentive compatibility), [Nissan, et al., 2007, 76-93], if there exists $(s_1^*(\theta_1), s_2^*(\theta_2), \dots, s_N^*(\theta_N))$ strategy vector for the $\Gamma = \left(\Theta_1, \Theta_2, \dots, \Theta_N, f(\cdot)\right)$ direct mechanism, which results in an equilibrium state, where $s_i^*(\theta_i) = \theta_i$ for all $\theta_i \in \Theta_i$ and i. Therefore, the social choice function is truthfully implementable if the truthful revelation of the type by the agents results in an equilibrium state of the mechanism $\Gamma = \left(\Theta_1, \Theta_2, \dots, \Theta_N, f(\cdot)\right)$. The truthful implementation or incentive compatibility of the first price sealed bid auction is evaluated in the analysis part of the article. First price sealed bid auction is not considered to be implementable by dominant strategies, therefore it cannot be truthfully implementable by dominant strategies. Although the first price sealed bid auction does not have a dominant equilibrium strategy, the solution to the problem may be found among Bayesian Nash equilibrium strategies. The strategy of the agent in first price sealed bid auction is dependent on the expectations of the strategies of the other agents, which in their turn depend on their types. The types of agents may be modeled by probability distribution. Based on the abovementioned a conclusion is made, that in first price sealed bid auction the strategies of agents are based on not actual utilities (which is the case under dominant strategies), but on expected utilities. In the framework of mechanism design, a strategy vector$(s_1^*(\theta_1), s_2^*(\theta_2), \dots, s_N^*(\theta_N))$ is recognized as a Bayesian Nash equilibrium strategy for a mechanism $\Gamma = \left(S_1, S_2, \dots, S_N, g(\cdot)\right)$, if for every agent i and for all types θ_i the expected utility from adopting the Bayesian Nash equilibrium strategy exceeds the expected utility from any other strategy [Börgers, et al., 2015, 76-93].

$$(1) \quad E_{\theta_{-i}}[u_i(g(s_i^*(\theta_i), s_{-i}^*), \theta_i)|\theta_i] \geq E_{\theta_{-i}}[u_i(g(s_i'(\theta_\mathrm{i}), s_{-i}^*), \theta_\mathrm{i})|\theta_i]$$

$for\ \forall\ i, \theta_i, s', s_{-i}$

In the context of first-price sealed bid auction, rational agents aim to maximize their expected utility. Since the expected utility from the implementation of Bayesian Nash equilibrium strategy is the highest, this strategy is deemed to be the one implemented by rational agents. The mechanism $\Gamma = \left(S_1, S_2, \dots, S_N, g(\cdot)\right)$ implements the social choice function f with Bayesian Nash equilibrium strategy, if there exists Bayesian Nash equilibrium strategy vector $(s_1^*(\theta_1), s_2^*(\theta_2), \dots, s_N^*(\theta_N))$ for the mechanism Γ, which brings the mechanism to the equilibrium state, where the result function is equal to the social choice function $g\left(s^*(\theta)\right) = f(\theta)$ for all $\theta \in \Theta$. If there exists Bayesian Nash equilibrium strategy for the first price sealed bid auction, then the mechanism implements a social choice function [Kephard, et al., 2006, 296-334].

The social choice function is considered truthfully implementable by Bayesian Nash equilibrium strategy (incentive compatible) if the strategy $s^*(\theta) = \theta$ is considered Bayesian Nash equilibrium strategy for a mechanism $\Gamma = \left(\Theta_1, \Theta_2, \dots, \Theta_N, f(\cdot)\right)$. According to the revelation principle, if there exists a $\Gamma = \left(S_1, S_2, \dots, S_N, g(\cdot)\right)$ mechanism, which implements the social choice function f in Bayesian Nash equilibrium, then the social choice function f is truthfully implementable by Bayesian Nash equilibrium.

Scientific novelty. This article utilizes the methodology and tools of mechanism design theory to dissect the first price sealed-bid auction. It presents a general solution applicable to any number of participants and calculates key auction performance indicators. Additionally, it assesses the impact of increasing the number of agents on various auction metrics. These contributions underscore the scientific novelty of the article, showcasing its addition to the existing body of knowledge on auction theory and mechanism design.

Analysis. In the first price sealed bid auction the participants reveal their bids simultaneously. The product is sold to the participant, who revealed the highest bid, and the participant pays that bid. The first price sealed bid auction with one indivisible good is considered in the article. As mentioned above, the authors have already given the particular solution to the auction for two participants, the conclusions of which are given in the table below.

Table 1. Conclusions regarding first price sealed bid auction with 2 agents

First price sealed bid auction	
Number of participants	2
Bayesian Nash equilibrium strategy	$\frac{\theta_i}{2}$
Equilibrium state	$y_1(\theta) = 0$ եթե $\theta_1 < \theta_2$ $y_2(\theta) = 1$ եթե $\theta_2 > \theta_2$ $y_1(\theta) = 0$ եթե $\theta_2 \leq \theta_1$ $t_1(\theta) = -\frac{1}{2}\theta_1 y_1(\theta)$ $t_2(\theta) = -\frac{1}{2}\theta_2 y_2(\theta)$
Implemented social choice function	$y_1(\theta) = 0$ եթե $\theta_1 < \theta_2$ $y_2(\theta) = 1$ եթե $\theta_2 > \theta_2$ $y_1(\theta) = 0$ եթե $\theta_2 \leq \theta_1$ $t_1(\theta) = -\frac{1}{2}\theta_1 y_1(\theta)$ $t_2(\theta) = -\frac{1}{2}\theta_2 y_2(\theta)$
The mechanism which truthfully implements the social choice function	$t_i = \frac{b_i y_i}{2}$
Probability of winning	θ_i
Expected utility of an agent	1/6 = 0.1667
Expected revenue of the seller	1/3 = 0.3333

In this article the set of agents $I = \{1,2, . . . , N\}$ is considered, which can be used to find the general solution to the auction not only for two agents, but for any number of agents. It is also viable to evaluate the impact of an increase in the number of participants on Bayesian Nash equilibrium strategies of the agents, equilibrium state, implemented social choice function, mechanism truthfully implementing the social choice function, probability of winning, expected utilities of the agents and expected revenue of the seller.

The X set of mutually exclusive alternatives are considered in the article: $x \in X$. In case of first price sealed bid auction the vector of alternative has the following formulation [Matsushima, 2007, 1-30].

(4) $X = \{(y_1, y_2, \ldots, y_N, t_1, t_2, \ldots, t_N): y_i = \{0,1\}$ *և* $t_i \in R$ *բոլոր i համար*, $\sum_i y_i = 1, \sum_i t_i \leq 0)\}$

The payments t_i made by an agent in an auction is determined by the formula:

(5) $t_i = b_i y_i$,

where b_i represents the bid submitted by an agent. The factor If the agent wins the auction y_i indicates the outcome of the auction for the agent: if the agent wins the auction ($y_i = 1$), they are required to pay their bid $t_i = b_i \times 1 = b_i$. Conversely, if the agent does not win the auction ($y_i = 0$), no payment is made $t_i = b_i \times 0 = 0$.

The utility function of an agent depends on θ_i type, while Θ_i denotes the set of all possible types. θ denotes the vector of types of all agents: $\theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_N)$. It is assumed that these types are random variables following a continuous uniform distribution) [Kuipers, et al., 2002, 1-85], [Forbes, et al., 2011, 176-182]. θ_i types are normalized over the interval [0,1]. The probability density function and cumulative distribution function of continuous uniform distribution are given bellow:

(6) $$f(\theta_i) = \begin{cases} \frac{1}{b-a} = \frac{1}{1-0} = 0.04, \textit{երբ } 0 \leq \theta_i \leq 1 \\ 0, \textit{երբ } 1 \geq \theta_i \textit{ կամ } 1 \leq \theta_i \end{cases}$$

(7) $$F(\theta_i) = \begin{cases} 0, \textit{երբ } 1 \geq \theta_i \\ \frac{\theta_i - 0}{1-\theta_i}, \textit{երբ } 0 \leq \theta_i \leq 1 \\ 1, \textit{երբ } 1 \leq \theta_i \end{cases}$$

In the framework of a first-price sealed bid auction, the mechanism can be represented as follows, considering N number of agents, the set of strategies consists of N objects: $\Gamma = \big(b_1(\theta_1), b_2(\theta_2), \ldots, b_N(\theta_N), g(\cdot)\big)$. Each agent i observes their own type θ_i (it is a private information known only to them) and places a bid according to b_i function. The problem of an agent in first price sealed bid auction is to maximize their expected utility by choosing a strategy function b_i:

(8) $$\max((\theta_i - b_i(\theta_i))F(b_i(\theta_i))$$

(9) $$F(b_i(\theta_i) = Prob(b_i \geq b_{-i})$$

(10) $$\begin{matrix} \max((\theta_1 - b_1(\theta_1))F(b_1(\theta_1)) \\ \max((\theta_2 - b_2(\theta_2))F(b_2(\theta_2)) \\ \ldots \\ \max((\theta_N - b_N(\theta_N))F(b_N(\theta_N)) \end{matrix}$$

The agent i chooses a strategy b_i to maximize the multiple of utility and the probability of winning. The problem of an agent is an optimization problem, which is formulated bellow, and is solved by the first order differentiation:

(11) $$\max((\theta_i - b_i(\theta_i))F(b_i(\theta_i)^{N-1}) = \max((\theta_i - b_i(\theta_i))(\frac{b_i(\theta_i)-0}{1-0})^{N-1})$$

(12) $$(\theta_i b_i(\theta_i)^{N-1} - b_i(\theta_i) b_i(\theta_i)^{N-1})' = (\theta_i b_i(\theta_i)^{N-1} - b_i(\theta_i)^N)'$$

(13) $(N-1)\theta_i b_i(\theta_i)^{N-2} - N b_i(\theta_i)^{N-1} = 0$

(14) $b_i(\theta_i) = \frac{(N-1)\theta_i}{N}$

In the context of an auction with N agents the Bayesian Nash equilibrium strategy is given by $\frac{(N-1)\theta_i}{N}$. . Notably, an auction involving two agents represents a specific instance of this general formula. When $N = 2$ the strategy simplifies to $\frac{(N-1)\theta_i}{N}$ simplifies to $\frac{\theta_i}{2}$, aligning with the previously identified Bayesian Nash equilibrium strategy for two-agent auctions. This analysis underscores a viable insight: as the number of agents increases, the bids submitted tend to be higher. The rationale behind this trend is that with more participants, the probability of winning the auction diminishes, which in turn reduces the expected utility for each agent. To counteract this decrease in expected utility, agents are motivated to place higher bids.

The relationship between the number of agents and their bidding strategies, with types normalized within the interval [0,1], is illustrated in Figure 1. It is clear from the figure that the disparity in bids intensifies for higher types. Figure 1 displays the bids of agents in auctions comprising 2, 3, 4, 5, and 6 agents, demonstrating the progression of bidding behavior as the auction becomes more competitive.

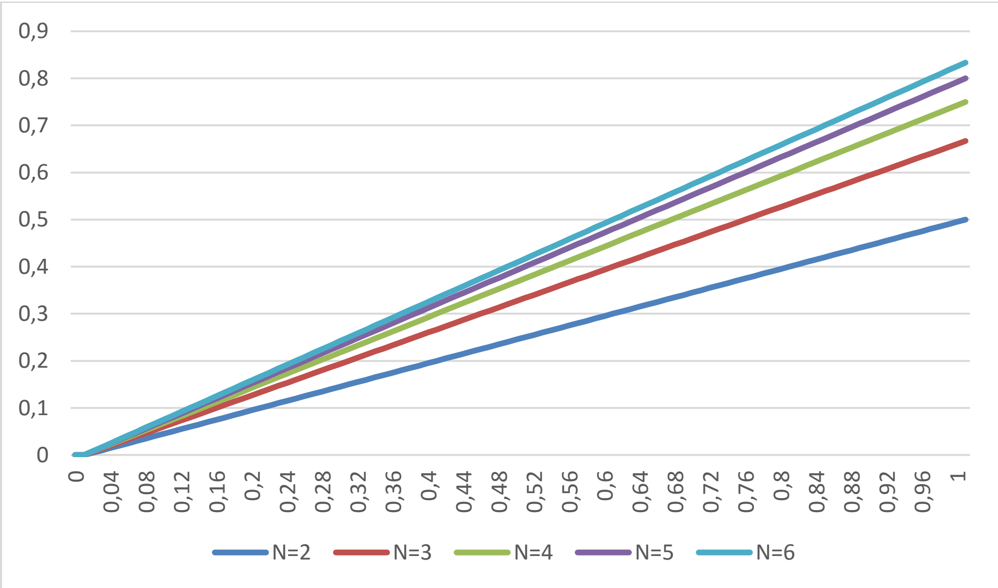

Figure 1. The placed bids in a first price sealed bid auction consisting of 2-6 participants

The bids placed in auctions featuring 10, 20, 30, 50, and 100 agents are illustrated in Figure 2. According to the analysis it can be stated that as the number of agents increases the Bayesian Nash equilibrium strategy increasingly aligns with incentive-compatible strategy. The implementation of the Bayesian Nash equilibrium strategy in the auction consisting of N agents results in an equilibrium state provided bellow:

$$(15)\ y_i(\theta) = 1 \text{ եթե } \theta_i \geq \max(\theta_{-i})$$
$$y_i(\theta) = 0 \text{ եթե } \theta_i < \max(\theta_{-i})$$
$$t_i(\theta) = -\frac{(N-1)}{N}\theta_i y_i(\theta)$$

Considering that the placed bids $-\frac{(N-1)}{N}\theta_i$ are higher in case of more participants, it can be stated that the increase in the number of agents will lead to increased payments. Therefore, the mechanism $\Gamma = \left(S_1, S_2, \dots, S_N, g(\cdot)\right)$ implements the social choice function f, which is represented by the formula (16) with Bayesian Nash equilibrium strategy. It is also evident, that the first price sealed bid auction consisting of N agents is not incentive compatible. More particularly, the agent having the type θ_i prefers not to reveal his true type θ_i, but another $-\frac{(N-1)}{N}\theta_i$ type, which results in higher expected utility.

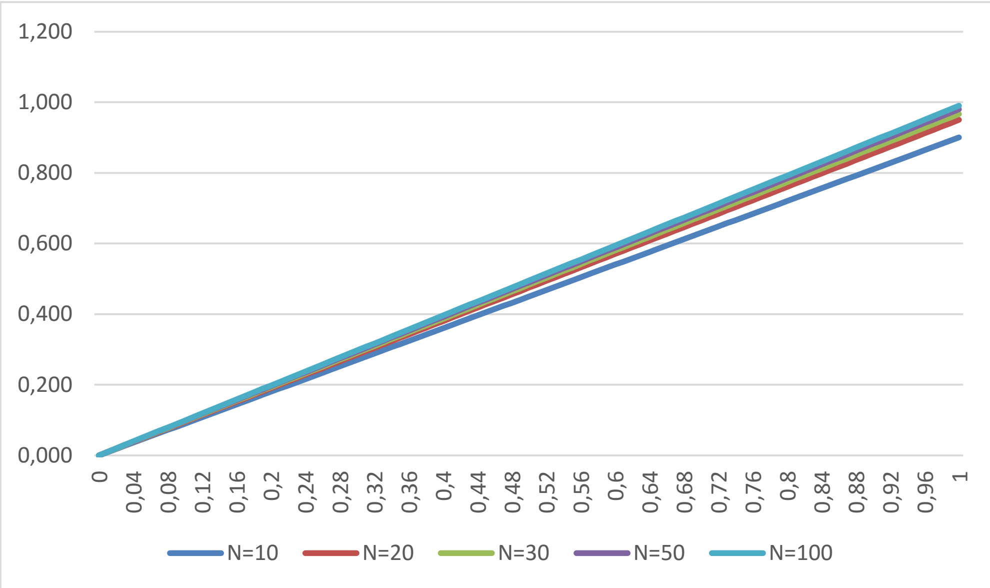

Figure 2. The placed bids in a first price sealed bid auction consisting of 10, 20, 30, 50, 100 participants

If the payment function of the auction, initially defined as $t_i = b_i y_i$, is modified to $t_i(\theta) = -\frac{(N-1)}{N} b_i y_i(\theta)$, then the mechanism $\Gamma = \left(S_1, S_2, \dots, S_N, g(\cdot)\right)$ achieves truthful implementation. Particularly, the mechanism truthfully implements the social choice function represented in formula (16).

In the end it is also viable to evaluate the impact of an increase of the number of agents in the auction on the expected utilities of the agents and on the expected revenue of the seller from the auction. Having the type θ_i the agent i places a bid and in case of winning pays the price $-\frac{(N-1)\theta_i}{N}$. As the types are random variables with continuous uniform distribution, the expected utility of an agent is calculated by integrating the utility function and the cumulative distribution function over the interval [0,1].

$$(16)\ \int_0^1 (\theta_i - \frac{(N-1)\theta_i}{N}) F(\theta_i)\,dx = \int_0^1 (\theta_i - \frac{(N-1)\theta_i}{N})\theta_i^{N-1} = \int_0^1 \left(\theta_i^{N} - \frac{(N-1)\theta_i^{N}}{N}\right)$$

$$(17)\ \int_0^1 \left(\theta_i^{N} - \frac{(N-1)\theta_i^{N}}{N}\right) = \frac{1^{N+1}}{N+1} - \frac{(N-1)1^{N+1}}{(N+1)N} = \frac{1}{N^2+N}$$

As indicated by formula (18), an increase in the number of agents leads to a reduction in expected utilities. This outcome is directly attributable to the adjusted Bayesian Nash Equilibrium strategy. As the probability of winning decreases with more participants, agents are compelled to submit higher bids. Consequently, winners end up paying a higher price, which, while increasing the cost of winning, diminishes the overall expected utility for each participant.

The seller's expected revenue from the winning agent can be represented by the price $\frac{(N-1)\theta_i}{N}$. Accordingly, the total expected payment to the seller from all participating agents can be expressed as:

$$(18)\ s = -\sum_{i=1}^{N} t_i(\theta) = \sum_{i=1}^{N} \frac{(N-1)}{N} \theta_i y_i(\theta)$$

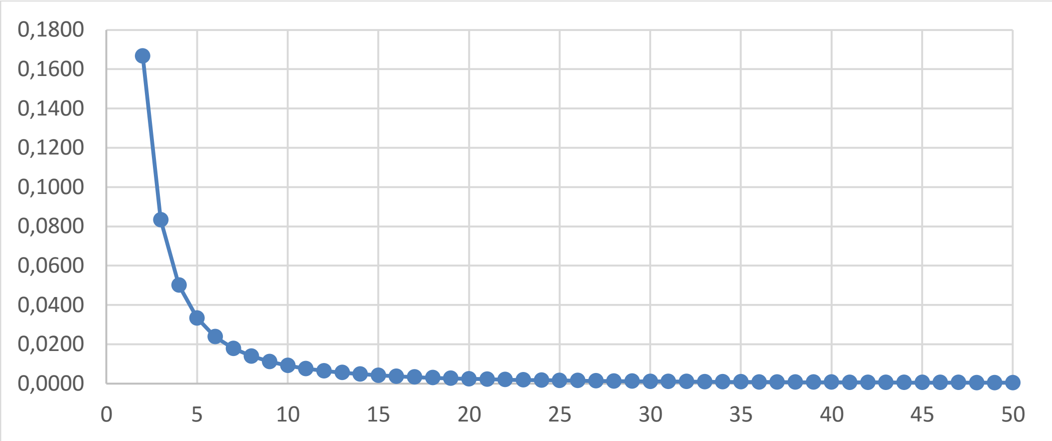

Figure 3. The expected utility of agents from first price sealed bid auction in case of 2-50 participants

As the types are random variables with a continuous uniform distribution, the expected revenue from the winning agent is derived by integrating the payment function $t_i(\theta)$ and the probability of winning in the interval [0,1]. The total expected revenue from the auction, therefore, is obtained by summing the revenues derived from each agent:

$$(19)\ s = \int_0^1 t_1(\theta_1)F(\theta_1)dx + \int_0^1 t_2(\theta_2)F(\theta_2)dx + \cdots + \int_0^1 t_i(\theta_i)F(\theta_i)dx$$

$$(20)\ s = \int_0^1 \frac{(N-1)}{N}\theta_1 F(\theta_1) + \int_0^1 \frac{(N-1)}{N}\theta_2 F(\theta_2)\,dx + \cdots += \int_0^1 \frac{(N-1)}{N}\theta_1^{N-1}\,dx + \int_0^1 \frac{(N-1)}{N}\theta_2^{N-1}\,dx + \cdots + \int_0^1 \frac{(N-1)}{N}\theta_N^{N-1}\,dx$$

$$(21)\ \int_0^1 \frac{(N-1)}{N}\theta_1{}^N = \frac{(N-1)}{N^2+N} \rightarrow s = \sum_{i=1}^{N} \frac{(N-1)}{N^2+N} = \frac{N(N-1)}{N^2+N}$$

As the number of participating agents increases, the expected revenue generating from each individual agent diminishes, but the total revenue from the auction exhibits an upward trend. Figure 3 presents the variation in expected utility for auctions with 2 to 50 agents, Figure 4 depicts the expected revenue of the seller for auctions with 2 to 50 agents. Finally, the table 2 compiles all the conclusions associated with the first price sealed bid auction involving N agents.

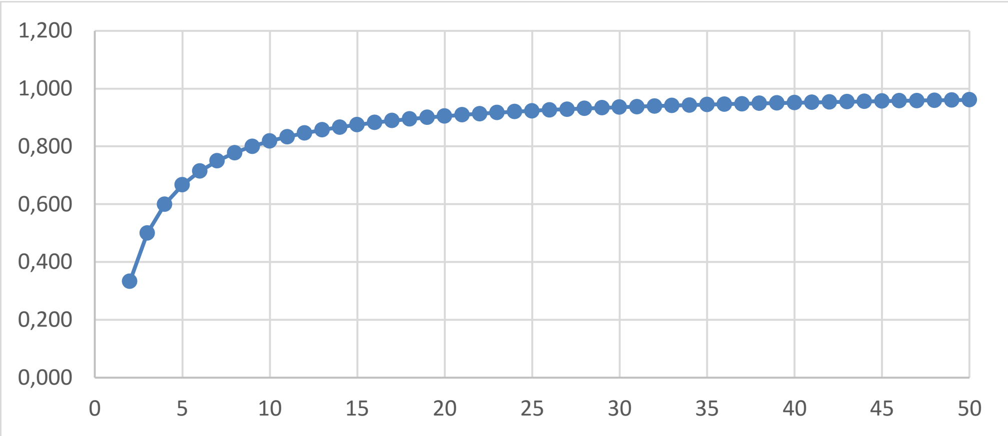

Figure 4. The expected revenue of the seller from the first price sealed bid auction in case of 2-50 agents.

Conclusions. Based on your detailed examination of the first-price sealed bid auction with varying numbers of agents, we can articulate several key conclusions.

- In an auction of N agents the Bayesian Nash equilibrium strategy is $\frac{(N-1)\theta_i}{N}$, and the increase in number of participants, results in increased bids placed by the agents according to the Bayesian Nash equilibrium strategy.
- In an auction of N agents the equilibrium state resulting from the implementation of Bayesian Nash equilibrium strategies is provided in the formula (16), additionally an increase of the number of agents results to an increase of the payments.
- In an auction of N agents to ensure the truthful implementation of the social choice function the transfer function $t_i = b_i y_i$ should be modified to $t_i = \frac{(N-1)b_i}{N} y_i$.
- In an auction of N agents the probability of winning of an agent is $\theta_i{}^{N-1}$.
- In an auction of N agents the expected utility of an agent is $\frac{1}{N^2+N}$
- In an auction of N agents the expected revenue of a seller is $\frac{N(N-1)}{N^2+N}$
- The increase in the number of agents results in:
  - increased bids,
  - reduced expected utilities of agents,

○ increased expected revenue of the seller.

Table 2. Conclusion regarding first price sealed bid auction consisting of N agents

First price sealed bid auction	
Number of participants	N
Bayesian Nash equilibrium strategy	$\frac{(N-1)\theta_i}{N}$
Equilibrium state	$y_i(\theta) = 1 \text{ եթե } \theta_i \geq \max(\theta_{-i})$ $y_i(\theta) = 0 \text{ եթե } \theta_i < \max(\theta_{-i})$ $t_i(\theta) = -\frac{(N-1)}{N}\theta_i y_i(\theta)$
Implemented social choice function	$y_i(\theta) = 1 \text{ եթե } \theta_i \geq \max(\theta_{-i})$ $y_i(\theta) = 0 \text{ եթե } \theta_i < \max(\theta_{-i})$ $t_i(\theta) = -\frac{(N-1)}{N}\theta_i y_i(\theta)$
The Mechanism which truthfully implements the social choice function	$t_i = \frac{(N-1)b_i}{N} y_i$
Probability of winning	${\theta_i}^{N-1}$
Expected utility of agent	$\frac{1}{N^2+N}$
Expected revenue of seller	$\frac{N(N-1)}{N^2+N}$

Recommendations:

- The agent types considered in this study are modelled as random variables following a continuous uniform distribution. For a more comprehensive understanding of bidding behaviour and auction dynamics, it is recommended to explore the impact of alternative distribution types on the model's variables. Analysing how different distributions affect bidding strategies, expected utilities, and the seller's expected revenue could provide deeper insights into the mechanisms underlying auction markets.
- To corroborate the findings presented in this article, conducting empirical analysis through surveys or scientific experiments is advised. Such empirical studies could involve collecting real world data from actual auctions or simulating auction environments to observe and analyse bidding behaviour and auction outcomes in real-world settings.
- The analytical framework and conclusions derived from this study offer a basis for predicting the outcomes of first price sealed bid auctions across varying numbers of agents.

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The analysis of first price single bid auction with uniformly distributed types within the scope of mechanism design theory

Key words: mechanism design theory, mechanism, auction, first price sealed bid auction, dominant strategy, Bayesian Nash equilibrium, incentive compatibility, social choice function.

In the article the first price sealed bid auction has been analysed within the scope of mechanism design theory. The sealed bid auction consisting of N number of agents has been observed finding the general solution to the auction. Bayesian Nash equilibrium strategies, the equilibrium state, and the implemented social choice function for the auction have been evaluated. The mechanism has been designed to truthfully implement the social choice function for the auction consisting of N agents. The variables such as the probability of an agent to win, the expected utility of an agent and the expected revenue of the seller have been evaluated. The effects of the increase of the number of agents on the variables of the model such as bids, Bayesian Nash equilibrium strategies, expected utility of an agent, expected revenue of the seller have been evaluated.